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OPTIMUM REINFORCEMENT BY TWO LONGITUDINAL STIFFENERS OF A PLATE SUBJECTED TO PURE BENDING

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Abstract—The paper examines the stability of a web reinforced by any number of unsymmetrically placed longitudinal stiffeners when subjected to pure bending. Solutions are presented for the cases where both of the longitudinal edges are assumed to be either simply supported or rigidly clamped. In both cases it is assumed that the edges of the panel bounded by the transverse stiffeners are simply supported.

Numerical solutions are presented for the case of two longitudinal stiffeners only. It is shown that when the longitudinal edges are clamped, the most effective positions for the stiffeners are at 0·136 d and 0·284 d from the compression flange, when a value of the buckling coefficient K equal to 356 is obtained. When the longitudinal edges are simply supported the corresponding values are 0·123 d, 0·275 d and 313 respectively.

For these two optimum conditions, the relationships between the stiffener parameters γ and β and the aspect ratio α have been determined. For values of α less than one, it is found that when both stiffeners are identical, the increase in longitudinal edge support from a simple pin joint to a clamped joint has very little influence upon the stiffener rigidity required to provide the maximum possible resistance to buckling. However, for values of α greater than one, the influence of the longitudinal edge support becomes increasingly significant.

NOTATION

d	panel depth
b	panel length
t	plate thickness
μ	Poisson's ratio
I_q	second moment of area of qth stiffener
γ_q	ratio of flexural rigidity of the qth longitudinal stiffener to flexural rigidity of plate, $= EI_q/Dd$
Δ_q	area of the qth longitudinal stiffener
β_q	ratio of area of qth longitudinal stiffener to cross-sectional area of panel, $= \Delta_q/dt$
D	flexural rigidity of unit width of plate, $= Et^3/12(1-\mu^2)$
α	aspect ratio of panel, $= b/d$
σ_c	compressive stress at edge of plate, $= K\pi^2 D/d^2 t$
$\sigma_\xi, \tau_{\xi\eta}, \sigma_\eta$	stress components for plate
ξ, η	Cartesian co-ordinates
x, y	$\pi\xi/b$, $\pi\eta/d$
w	transverse deflexion of plate's middle surface
η_q	value of η at qth longitudinal stiffener
χ	$\sigma_c b^2 t/\pi^2 D = K\alpha^2$
P	number of longitudinal stiffeners

1. INTRODUCTION

It is now generally accepted that the economical design of a deep plate girder results in the use of a thin web reinforced by a system of transverse and longitudinal stiffeners. When the loading is one of pure bending it will be necessary to employ a number of longitudinal stiffeners in the compression zone. Although satisfactory solutions are available for the buckling of a web reinforced by a single longitudinal stiffener, no complete solution has been obtained for the buckling of such webs when reinforced by two or more longitudinal stiffeners. In the present paper a general solution to this problem is presented, although numerical results are only provided for the case of a web reinforced by two longitudinal stiffeners.

The structure considered is shown in Fig. 1, which represents a panel of longitudinally and transversely stiffened web plate. It is assumed that the flanges provide either a simple pin support or a rigid support along the longitudinal edges OA and BC whilst the transverse stiffening results in a simple support being provided to the panel along edges OB and AC. MN, PQ, RS are longitudinal stiffeners. The load applied to the panel is assumed to vary linearly along OB and AC from a compressive stress at O and A to a tensile stress of the same magnitude at B and C. The stress system to which this gives rise is:

$$\sigma_\xi = -(1-2\eta/d)\sigma_c, \qquad \tau_{\xi\eta} = 0, \qquad \sigma_\eta = 0. \tag{1}$$

In this solution it is assumed that the longitudinal stiffeners are symmetrically placed about the mid-plane of the web and that their torsional rigidity can be ignored. The problem is to determine the optimum placing of the stiffeners and to determine the relationships which exist between the non-dimensional parameters γ, β, α and the buckling stress parameter K.

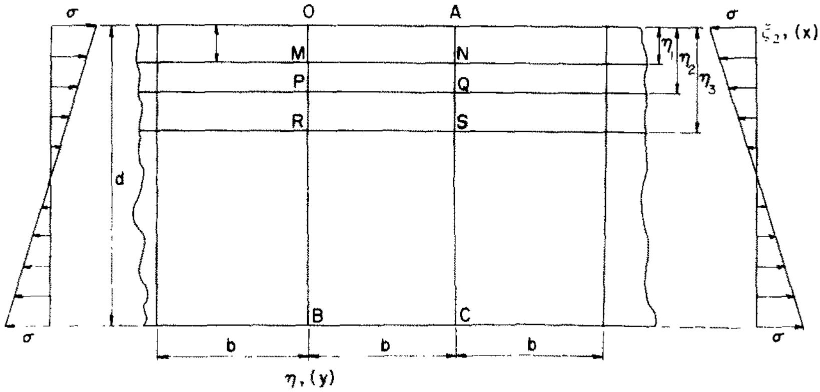

FIG. 1

2. HISTORICAL SURVEY

In a recent paper, Leggett and Rockey [1] have presented a survey of the previous work which deals with the buckling of webs subjected to pure bending, therefore only a brief survey will be given in this Section. In 1960, Massonnet *et al.* [2] presented the results of an extensive study of the buckling of a web clamped along its longitudinal edges and reinforced by a single longitudinal stiffener. The results of their investigation are in reasonable agreement with the solutions obtained by Leggett and Rockey [1], and Ceradini [3], for the case of a torsionally weak longitudinal stiffener, and with that obtained by Rockey [4] when the torsional rigidity of the longitudinal stiffener is also allowed for.

Richmond [5] has recently presented an approximate method for computing the buckling stress of simply supported plates reinforced by symmetrically disposed stiffeners. In his approach, Richmond considers the longitudinally stiffened plate to behave like an orthotropic plate. As a consequence of his method of solution, Richmond does not determine the limiting value of K corresponding to the stiffeners being nodal. Although Richmond presented an approximate method for dealing with unsymmetrically placed stiffeners, he did not give any detailed results obtained using this method.

In a series of papers [6, 7] Klöppel and Scheer have given solutions for simply supported rectangular plates under combined bending and thrust, for the cases of a single longitudinal stiffener or two equal longitudinal stiffeners.

3. THEORETICAL SOLUTION

In reference [1], Leggett and Rockey presented a theoretical solution to the buckling under pure bending of a rectangular plate clamped along its longitudinal edges, when it is reinforced by a single longitudinal stiffener. In the present paper, this solution is extended to deal with the case of a number of longitudinal stiffeners. In addition a solution is provided for the case when the longitudinal edges are simply supported.

To save undue repetition only an outline of these solutions is presented here.

3.1. *Longitudinal edges clamped*

The deflection function employed, which completely satisfies the boundary conditions, is given by

$$w = \sin x \sum_{m=1}^{\infty} A_m \phi_m(y), \tag{2}$$

where

$$\phi_m(y) = \sin my - my + \frac{m}{\pi}[2+(-1)^m]y^2 - \frac{m}{\pi}[1+(-1)^m]y^3. \tag{3}$$

If η_q denotes the position of the qth longitudinal stiffener from the compression flange and β_q the area ratio of this stiffener, then the final energy equation obtained for the stiffened plate–see equation (17) of [1], is given by

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}A_mA_n - \chi \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} d_{mn}A_mA_n$$
$$+\pi \sum_{q=1}^{P} \{[\gamma_q - \chi\beta_q(1-2\eta_q/d)][\sum_{m=1}^{\infty} A_m Z_{mq}]^2\} = 0, \tag{4}$$

where

$$Z_{mq} = \phi_m(\pi\eta_q/d) \tag{5}$$

$$d_{mn} = a_{mn}(\alpha = 0) - \frac{2}{\pi} b_{mn}, \tag{6}$$

a_{mn} and b_{mn} being defined in equations (12) (13) and (16) of [1].

Since the plate is in neutral equilibrium, equation (4) must remain true for any small variations in the form of distortion. Thus differentiating equation (4) partially with respect to A_m gives:

$$\sum_{n=1}^{\infty} a_{mn}A_n - \chi \sum_{n=1}^{\infty} d_{mn}A_n + \pi \sum_{q=1}^{P} \left[\gamma_q - \chi\beta_q\left(1 - \frac{2\eta_q}{d}\right)\right]\left[Z_{mq} \sum_{n=1}^{\infty} Z_{nq}A_n\right] = 0. \tag{7}$$

Since the coefficients A_n, cannot be zero, the condition that equation (7) shall possess a non-trivial solution is

$$\det\left(a_{mn}-\chi d_{mn}+\pi\sum_{q=1}^{P}\left[\gamma_q-\chi\beta_q\left(1-\frac{2\eta_q}{d}\right)\right]Z_{mq}Z_{nq}\right)=0. \tag{8}$$

If the stiffeners have sufficient rigidity they will remain straight and the plate will buckle between them. Whilst such a deflexion is capable of representation by the series (2), it is nevertheless easier to determine the critical load in the following way.

The condition that the deflexion along the stiffeners is zero is that the following equations should be satisfied:

$$\sum_{n=1}^{\infty} A_n Z_{nq} = 0 \qquad q = 1, 2, \ldots, P. \tag{9}$$

Obviously, since the stiffeners do not deflect, those energy terms involving γ_q and β_q disappear. Using the Lagrange Multiplier method, as in [1], one obtains the following final equations:

$$\sum_{n=1}^{\infty}(a_{mn}-\chi d_{mn})A_n+\sum_{q=1}^{P}\lambda_q Z_{mq}=0. \tag{10}$$

The condition that the equations (9) and (10) shall possess a non-trivial solution for the A_n is *

$$\begin{vmatrix} \mathbf{a}-\chi\mathbf{d} & \mathbf{Z} \\ \mathbf{Z}^T & \mathbf{O} \end{vmatrix} = 0, \tag{11}$$

where **a**, **d** and **Z** denote the matrices with elements a_{mn}, d_{mn} and Z_{mq} respectively, and $\mathbf{Z}^T$ denotes the transpose of **Z**.

The 'nodal' value K_L of K obtained from equation (11) is the maximum obtainable with the given stiffener positions.

3.2. *Longitudinal edges simply supported*

In this case, the deflexion function used is given by

$$w = \sin x \sum_{m=1}^{\infty} A_m \sin my. \tag{12}$$

Proceeding as in Section 3.1, equations (7) and (8) are again obtained, where in this case

$$\begin{aligned} a_{mn} &= \frac{\pi}{2}(1+\alpha^2 m^2)^2 && \text{if } m=n \\ &= 0 && \text{otherwise,} \\ d_{mn} &= \frac{4mn}{(m^2-n^2)^2} && \text{if } (m+n) \text{ is odd} \\ &= 0 && \text{otherwise,} \\ Z_{nq} &= \sin(n\pi\eta_q/d). \end{aligned} \tag{13}$$

* The determinant given is in fact the condition for a non-trivial solution for the A_n and the λ_q. However any non-trivial solution in which all the A_n vanish would give $\sum_{q=1}^{P}\lambda_q Z_{mq}=0$, which cannot occur since the Z_{mq} are linearly independent.

3.3. *Relationship between stiffener rigidities*

Having determined the 'nodal' value K_L of K, as described in Section 3.1, for the given stiffener positions, it is necessary to determine the relationship between $\gamma_1, \gamma_2, \ldots \gamma_P$; $\beta_1, \beta_2, \ldots, \beta_P$ and α which ensures that the buckling load $K = K_L$ is achieved. For given values of K and $\eta_1, \eta_2, \ldots, \eta_p$ equation (8) gives a single functional relationship between the $(2P+1)$ parameters γ_q, β_q and α. In general, the determination of a functional relationship between many parameters is very complicated, but in this case it can be shown that the relationship is of a very simple kind and is completely determined by 2^P-1 functions of α.

First we investigate the relationship between γ and β for a particular stiffener. Let γ_q^* be the value of γ_q when $\beta_q = 0$. It follows immediately from (8) that

$$\gamma_q^* = \gamma_q - K\alpha^2[1-2\eta_q/d]\beta_q, \tag{14}$$

so that

$$\gamma_q = \gamma_q^* + K\alpha^2(1-2\eta_q/d)\beta_q. \tag{15}$$

The rigidity γ_q thus depends linearly on the area parameter β_q, and this determines completely the effect of β_q.

A similar relationship holds between γ and β in the case of one stiffener and may be verified in the results of Rockey and Leggett [1], and Stüssi *et al.* [8, 9].

The relationship between the γ_q^* will now be determined. Equation (8) may be written

$$\det\left(a_{mn} - \alpha^2 K d_{mn} + \pi \sum_{q=1}^{P} \gamma_q^* Z_{mq} Z_{nq}\right) = 0. \tag{16}$$

Now, for each q the matrix $Z_{mq}Z_{nq}$ is of rank 1 (each row being a multiple of the first row), so the determinant in (16) is linear in each γ_q^*. The relationship between the γ_q^* is, therefore, a multilinear form in which the coefficients are functions of α. A general multilinear form in P variables has 2^P-1 independent coefficients, the relationships between the γ_q^* taking the following forms in the cases of $P = 2$ and 3:

$$P = 2: \quad \gamma_1^*\gamma_2^* + a\gamma_1^* + b\gamma_2^* + c = 0, \tag{17}$$

$$P = 3: \quad \gamma_1^*\gamma_2^*\gamma_3^* + a\gamma_1^*\gamma_2^* + b\gamma_2^*\gamma_3^* + c\gamma_3^*\gamma_1^* + d\gamma_1^* + e\gamma_2^* + f\gamma_3^* + g = 0, \tag{18}$$

the coefficients $a, b, c, \ldots, g$ being functions of α, and depending also on $K, \eta_1, \eta_2, \ldots, \eta_P$.

In the case of two stiffeners, (17) may be written in the alternative form

$$(\gamma_1^* - \bar{\gamma}_1^*)(\gamma_2^* - \bar{\gamma}_2^*) = C^2. \tag{19}$$

Writing

$$\bar{\gamma}_q = \bar{\gamma}_q^* + K\alpha^2[1-2\eta_q/d]\beta_q, \tag{20}$$

equation (19) gives

$$(\gamma_1 - \bar{\gamma}_1)(\gamma_2 - \bar{\gamma}_2) = C^2. \tag{21}$$

Equations (20) and (21) give the desired relationship between γ_1, γ_2, β_1 and β_2 in terms of the three functions of α: $\bar{\gamma}_1^*$, $\bar{\gamma}_2^*$ and C. Equation (21) shows that for given β_1, β_2 and α the graph of γ_1 against γ_2 is a rectangular hyperbola with asymptotes $\gamma_1 = \bar{\gamma}_1$ and $\gamma_2 = \bar{\gamma}_2$. The significance of $\bar{\gamma}_1$ is that it is the rigidity required for the first stiffener when the second stiffener has infinite rigidity, that is to say when the second stiffener is nodal. It is worthy of emphasis that the linear relationship between γ_1 and β_1 and between γ_2 and β_2 and the hyperbolic relationship between γ_1 and γ_2 are *precise* and are not approximate.

The values of $\bar{\gamma}_1^*$ may be determined very simply by setting $\beta_1 = \beta_2 = 0$ and $\gamma_2 = 0$ in the energy equation (4) and carrying out variation subject to the single constraint

$$\sum_r A_r Z_{r2} = 0. \tag{22}$$

The resulting determinantal equation is again linear in $\bar{\gamma}_1^*$ and so $\bar{\gamma}_1^*$ may be determined by evaluation of a finite approximation to the determinant for two trial values of $\bar{\gamma}_1^*$. $\bar{\gamma}_2^*$ may be similarly determined. The 'constant' C may be determined by finding any other point on the rectangular hyperbola, that is to say by giving γ_2^* a value exceeding $\bar{\gamma}_2^*$ and by solving the determinantal equation for γ_1^*, again using the linearity. It was found, however, that it was difficult to choose *a priori* a value for γ_2^* which was large enough to make both $(\gamma_1^* - \bar{\gamma}_1^*)$ and $(\gamma_2^* - \bar{\gamma}_2^*)$ of the same order of magnitude, so that any small errors in the determination of γ_1^* and $\bar{\gamma}_1^*$ or of $\bar{\gamma}_2^*$ did not cause significant errors in C. Accordingly, C was found explicitly by setting

$$\begin{aligned} \gamma_1^* &= \bar{\gamma}_1^* + C \\ \gamma_2^* &= \bar{\gamma}_2^* + C \end{aligned} \tag{23}$$

in equation (16) and solving a finite approximation to the determinantal equation for C. This equation is, of course, not linear in C. It was found, however, that application of the method of inverse linear interpolation to the value of the determinant resulted in convergence after only a few iterations.

The beautiful simplicity of the linear and hyperbolic relations is complicated slightly by the fact that it is necessary to consider the case of a deflexion with two or more waves as well as the case of one wave. This is allowed for in the usual way by repeating the relationship already obtained at 2α, 3α and so on. The result is that in certain cases the relationship between γ_1 and γ_2 is not a single rectangular hyperbola, but a curve consisting of parts of two or more rectangular hyperbolae. It is clear that in equation (20) the 'fundamental' single half wave value of α must be used.

4. DISCUSSION OF RESULTS

Although the theory presented deals with any number of longitudinal stiffeners, numerical results are only presented for the case of two longitudinal stiffeners. Using equations (11) and (13), values of K_L, (i.e. the buckling coefficient K obtained with stiffeners of infinite rigidity) have been determined for many different placings of the stiffeners. From these values it has been possible to determine where the two stiffeners should be placed in order to yield the maximum buckling resistance.

Figure 2 gives some typical K_L, α relationships for a selection of stiffener positions. It will be seen that for certain of these, there are two separate minima; the case where the two stiffeners are at $0{\cdot}14\,d$ and $0{\cdot}28\,d$ is close to the optimum configuration, since the two values of K_L at the minima are close.

Figure 3 presents the collected set of results for the case where the longitudinal edges are clamped. Each point in Fig. 3 is the minimum value of K_L obtained from curves such as those in Fig. 2. The optimum placing of the two stiffeners was found from this

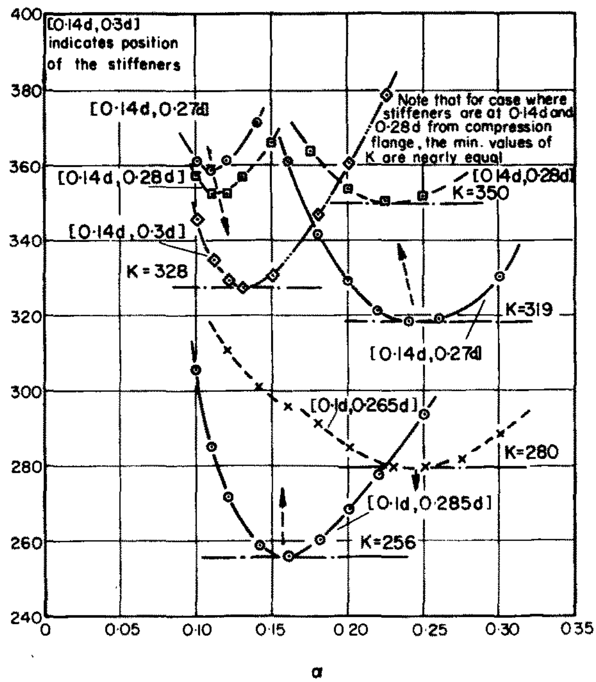

FIG. 2. Values of buckling coefficients K when both stiffeners do not deflect.

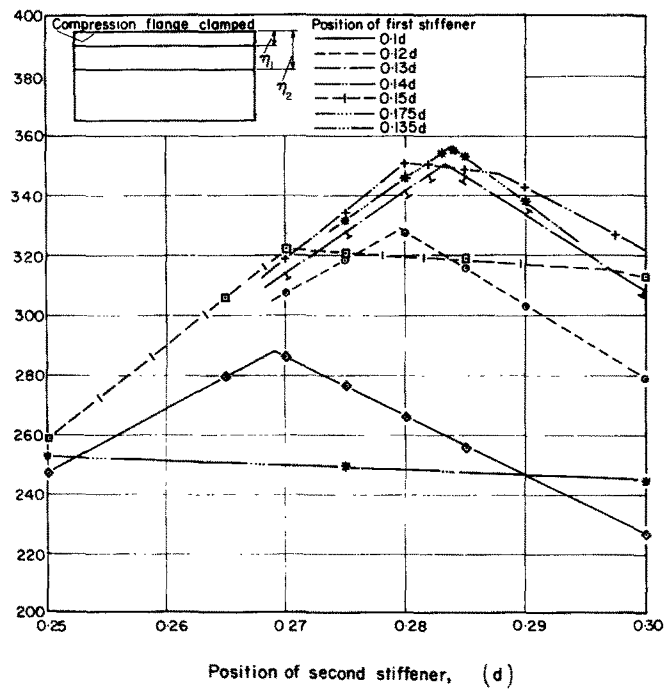

FIG. 3. Position of second stiffener.

study to be at 0·135 d and 0·284 d from the compression flange, when a value of K_L equal to 356 is obtained.

Table 1 gives the rate of convergence of the series employed. In the work presented 18 × 18 determinants were employed in the evaluation of K_L and 12 × 12 determinants in the determination of γ_1 and γ_2.

TABLE 1

K calculated–both stiffeners nodal (γ's = ∞) $\eta_1 = 0{\cdot}2\,d$ $\eta_2 = 0{\cdot}5\,d$ $\alpha = 0{\cdot}3$	Size of matrix	7 × 7	9 × 9	11 × 11	14 × 14	18 × 18
	K	211·55	197·11	195·89	195·56	195·44
K calculated–both stiffeners nodal (γ's = ∞) $\eta_1 = 0{\cdot}15$ $\eta_2 = 0{\cdot}60$ $\alpha = 0{\cdot}25$	Size of matrix	7 × 7	9 × 9	11 × 11	14 × 14	18 × 18
	K	117·875	113·81	112·990	112·031	111.831
Stiffener nodal at 0·13 d γ calculated for stiffener at 0·285 d $\beta = 0{\cdot}1$, $\alpha = 1{\cdot}0$, $K = 346$	Size of matrix	6 × 6	8 × 8	10 × 10	14 × 14	18 × 18
	γ	27·6679	27·8588	27·945	28·2044	28.2341
Stiffener nodal at 0·13 d γ calculated for stiffener at 0·25 d $\beta = 0{\cdot}1$, $\alpha = 0{\cdot}9$, $K = 260$	Size of matrix	6 × 6	8 × 8	10 × 10	14 × 14	18 × 18
	γ	18.4906	18·5746	18·704	18·8207	18·8649
Stiffener nodal at 0·25 d γ calculated for stiffener at 0·13 d $\beta = 0{\cdot}1$, $\alpha = 0{\cdot}5$, $K = 260$	Size of matrix	6 × 6	8 × 8	10 × 10	14 × 14	18 × 18
	γ	4·9716	4·9781	4·9854	4·9917	4·9928
Stiffener nodal at 0·5 d γ calculated for stiffener at 0.2 d $\beta = 0{\cdot}1$, $\alpha = 1{\cdot}0$, $K = 142{\cdot}6$	Size of matrix	7 × 7	10 × 10	12 × 12	14 × 14	16 × 16
	γ	8·120	8·220	8·235	8·241	8·246

Proceeding in a similar manner for simply supported longitudinal edges, the optimum placing of the two stiffeners was found to be at 0·123 d and 0·275 d from the compression flange when a value of K_L equal to 315 is obtained.

Relationships between the stiffener rigidities

Using equations (19)–(21) and the values of $\bar{\gamma}_1^*$, $\bar{\gamma}_2^*$ and C given in Table 2, it is possible to determine curves such as those given in Fig. 4 for the case where the longitudinal edges are simply supported and $\alpha = 0{\cdot}6$. Consider then the case where the two stiffeners have an area of 0·1 dt. Any values of γ_1 and γ_2 which can be obtained from the rectangular hyperbola ABCD will yield the maximum value of K_L of 313. If the area of the second stiffener is increased to 0·2 dt, then the relevant hyperbola would be EFCD.

Each hyperbola has, of course, two branches; however, only the outer branch is shown. With those values of γ_1 and γ_2 associated with the outer branch, a value of the buckling coefficient K equal to 313 will be obtained. A value of 313 will also be obtained with the inner branch, but in this case it is not the least positive value of K satisfying equation (8) and is therefore not of interest here.

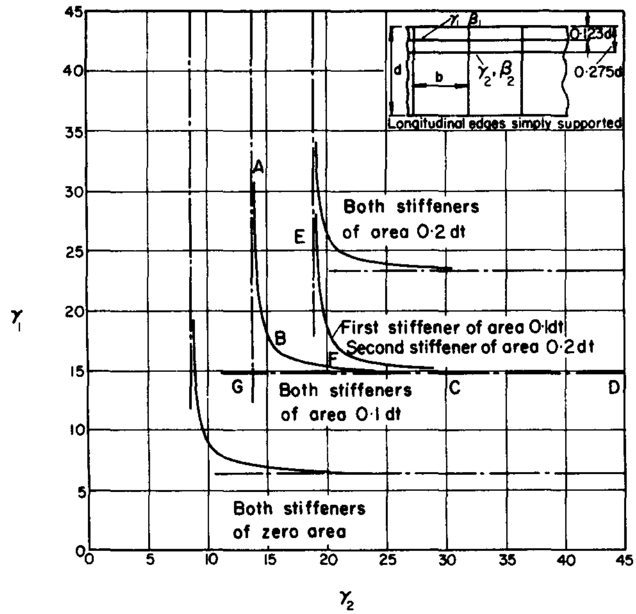

FIG. 4

Figure 5 gives the relationship between γ_1 and γ_2 when both stiffeners have zero area, for different values of the aspect ratio α. It will be noted that the relationship between γ_1 and γ_2 is given by a single hyperbola for the cases of α equal to 0·4, 0·6, 0·7, 0·8 and 0·9 but that for 1·0, 1·1 and 1·2, it consists of parts of two hyperbolae, this being

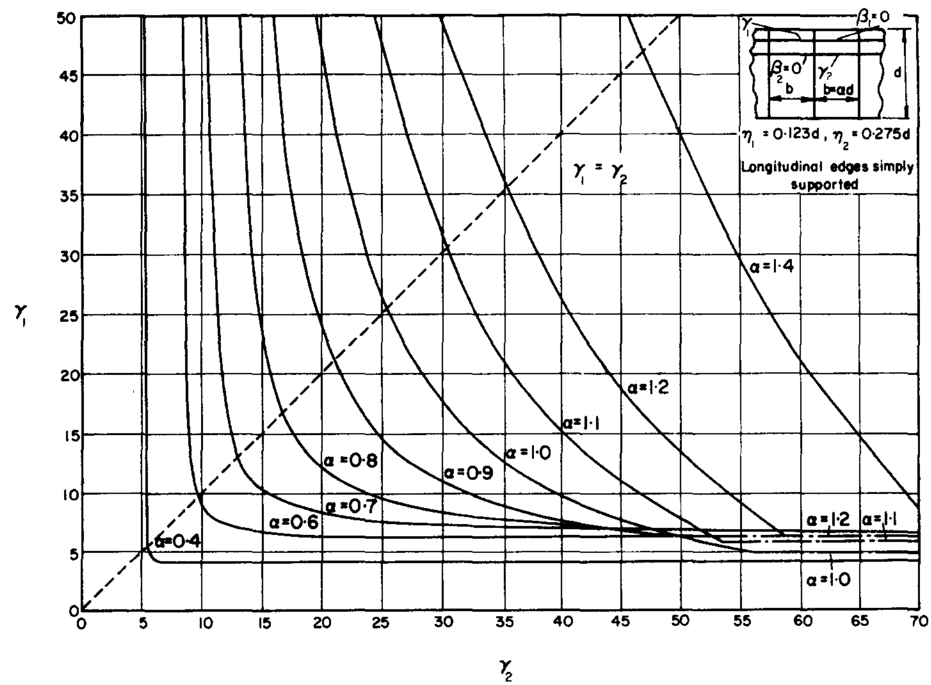

FIG. 5

caused by the consideration of more than one half wave as discussed in Section 3.3. Thus we see that by employing equations (19) and (21), in conjunction with Table 2, values of γ_1 and γ_2 can be calculated for any condition.

TABLE 2

	Longitudinal edges simply supported $K = 313$			Longitudinal edges clamped $K = 356$		
	$\eta_1 = 0{\cdot}123\,d$	$\eta_2 = 0{\cdot}275\,d$		$\eta_1 = 0{\cdot}136\,d$	$\eta_2 = 0{\cdot}284\,d$	
α	$\bar{\gamma}_1^*$	$\bar{\gamma}_2^*$	C	$\bar{\gamma}_1^*$	$\bar{\gamma}_2^*$	C
0·4	4·22	5·53	0·427	3·84	5·66	0·215
0·6	6·34	8·78	1·953	2·98	9·00	2·85
0·7	6·48	10.57	4.122	−0·06	10·77	5·69
0·8	5·35	12·17	7·345	−6·08	12·27	9·92
0·9	2·39	13·35	11·94	−16·16	13·23	15·96
1·0	−3·00	13·87	18·27	−31·54	13·40	24·27
1·1	−11·55	13·47	26·72	−53.56	12.48	35.37
1·2	−23·99	11·86	37·73	−83·72	10·14	49·81
1·3	−41·18	8·75	51·74	−123·7	6·04	68·22
1·4	−64·01	3·80	69·28	−175·2	−0·19	91·24
1·5	−93·44	−3·32	90·85	−240·1	−8·93	119·6
1·6	−130·5	−12·98	117·0	−320·6	−20·60	154·0
1·8	−232·1	−41·46	185·7	−536·9	−54·52	244·2
2·0	−378·5	−84·88	280·6	−843·3	−105·7	369·0
2·2	−580·2	−146·8	407·2	−1261	−178·4	535·9
2·4	−849·1	−231·0	572·0	−1814	−277·2	753·7
2·6	−1198	−341·6	781·8	−2527	−407·0	1032
2·8	−1641	−482·7	1044	−3430	−573·2	1380
3·0	−2192	−659·1	1365	−4551	−781·7	1808
3·2	−2869	−875·6	1756	−5924	−1039	2330
3·4	−3687	−1137	2224	−7582	−1351	2957
3·6	−4665	−1449	2777	−9564	−1725	3703
3·8	−5822	−1818	3428	−11910	−2169	4581
4·0	−7178	−2248	4186			
4·2	−8755	−2747	5061			
4·4	−10570	−3321	6066			
4·6	−12660	−3976	7211			
4·8	−15040	−4721	8512			

One design relationship which would clearly be convenient for both the designer and the fabricator, will be when the two stiffeners have the same size; i.e. $\gamma_1 = \gamma_2$ and $\beta_1 = \beta_2$. The dotted line in Fig. 5 gives this relationship. Figure 6 gives the relationships between the values of $\gamma_1(= \gamma_2)$ and the aspect ratio α for five values of β for the two longitudinal edge conditions considered. This yields a very interesting feature, namely that for values of α up to 1·0 there is very little difference between the values of γ required for the two different edge conditions. This will clearly facilitate the development of suitable design rules.

In the case of simply supported edges, the value of $\gamma_1(= \gamma_2)$ when $\alpha = 0{\cdot}6$ and $\beta_1 = \beta_2 = 0{\cdot}1$ is 16·36; see point B in Fig. 4. The K, α curves for stiffeners having these properties are given in Fig. 7, which shows quite clearly that the value of $K = 313$ is achieved.

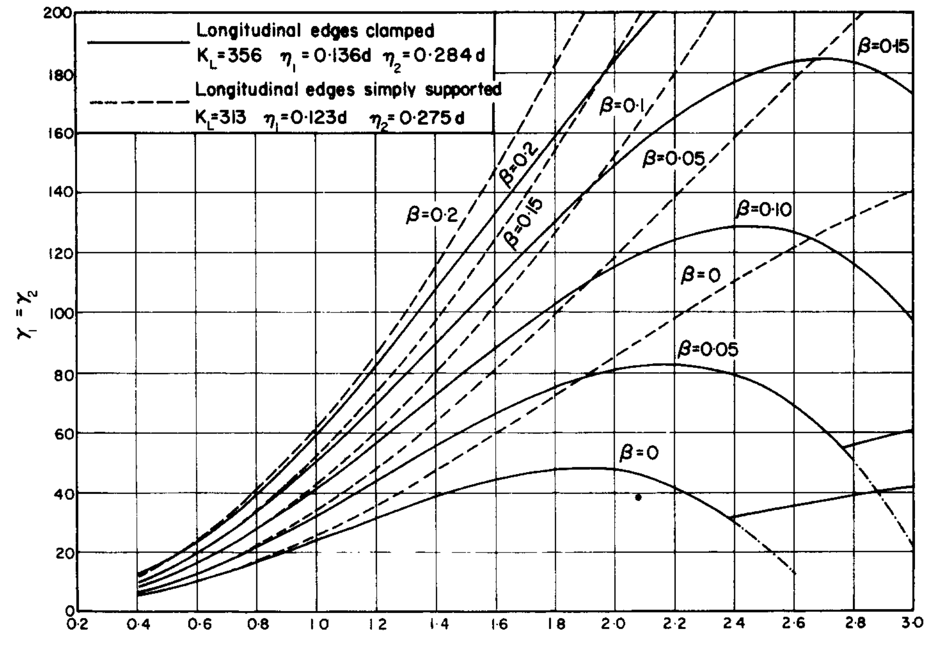

FIG. 6

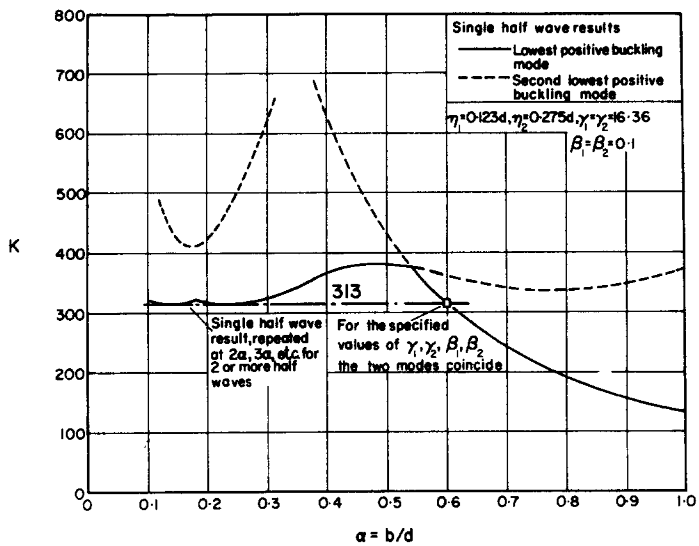

FIG. 7

One point which should be fully appreciated is that if the values of $\bar{\gamma}_1^*$ and $\bar{\gamma}_2^*$ for the case of $\beta_1 = \beta_2 = 0$, and $\bar{\gamma}_1$ and $\bar{\gamma}_2$ for other values of β, are used directly, then the value of K obtained will be less than the maximum value K_L. Referring to Fig. 4, for values of $\beta_1 = \beta_2 = 0{\cdot}1$, equation (21) yields the hyperbola ABCD, whilst direct application of

$\bar{\gamma}_1$ and $\bar{\gamma}_2$ from equation (20) would yield the values of γ_1 and γ_2 corresponding to point 'G'. It is therefore essential that the relationship given in equation (21) be used.

Figures 8, 9, 10 and 11 give values of $\bar{\gamma}_1$ and $\bar{\gamma}_2$ for three values of the area ratio β, for the two edge conditions considered. Using these curves and Table 2, in conjunction with equations (19)–(21), it is possible to determine the size of the longitudinal stiffeners in order that the plate shall provide the maximum buckling resistance.

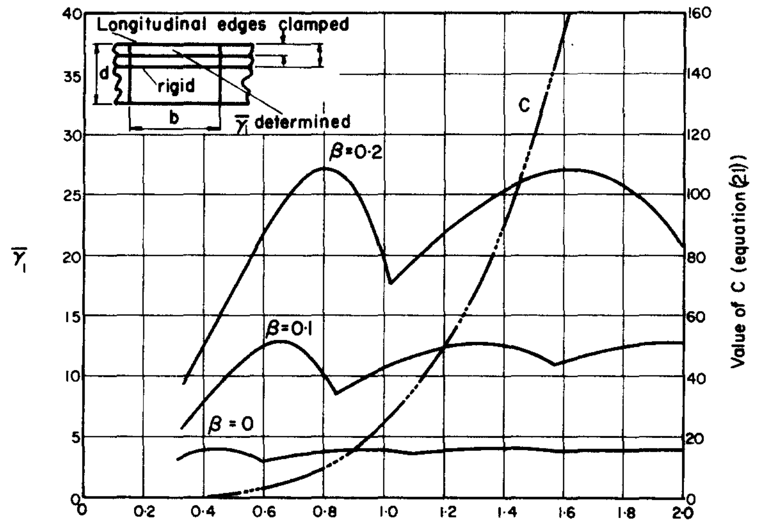

FIG. 8

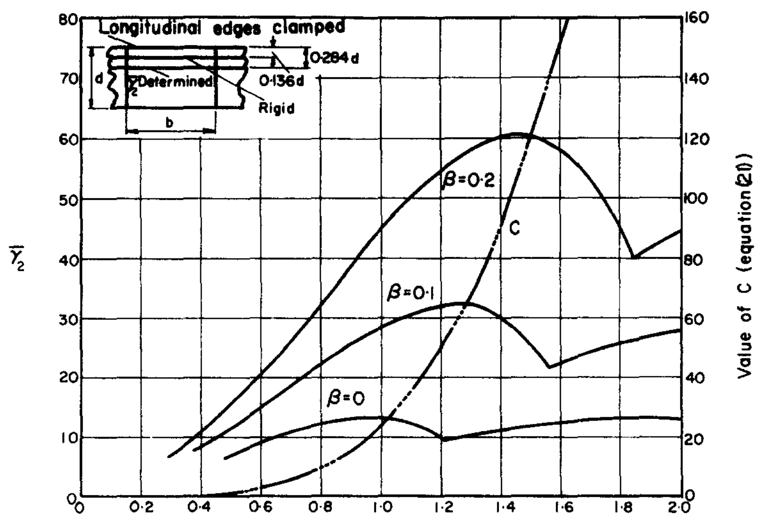

FIG. 9

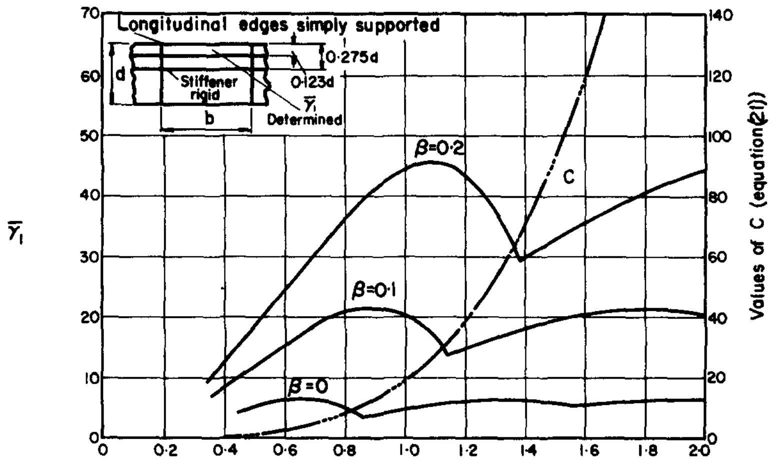

FIG. 10

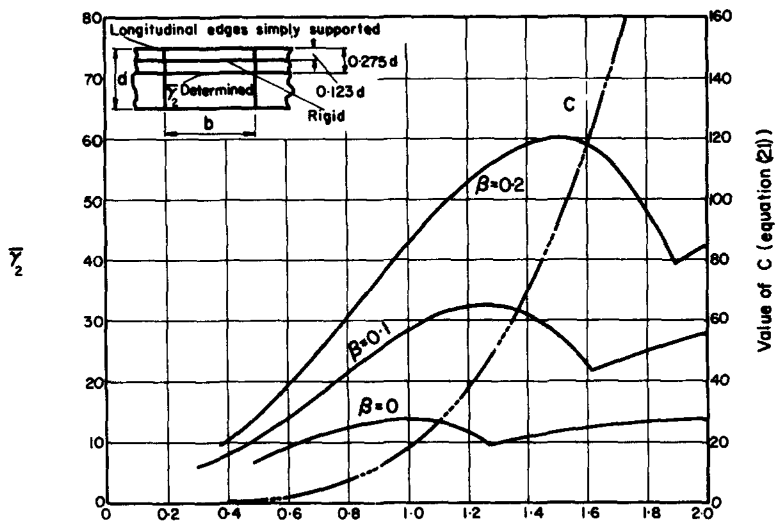

FIG. 11

CONCLUSIONS

The optimum positions for two longitudinal stiffeners on webs subjected to pure bending have been determined for the cases where both the longitudinal edges are either rigidly clamped or simply supported. For both of these conditions relationships between the flexural rigidity and area of the stiffeners and the aspect ratio of the panels have been determined.

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Zusammenfassung—Die optimale Lage von zwei Längsversteifungen für Platten die auf reine Biegung beansprucht werden ist bestimmt worden für den Fall, dass die beiden Längsseiten entweder starr eingespannt sind oder frei aufliegen. Für beide Fälle der Lagerung werden die Beziehungen zwischen der Biegesteifigkeit und Fläche der Versteifungen und dem Seitenverhältnis der Platten aufgestellt.

Абстракт—В работе рассматривается устойчивость стенки, укрепленной каким угодно числом несимметрично расположенных продольных уголков жесткости, подвергнутой чистому сгибанию. Даются решения для случаев в которых оба продольных края принимаются как либо свободно опирающиеся, либо жестко зажатые. В обоих случаях принимается, что края панели ограниченные поперечными уголками жесткости являются свободно опирающимися.

Численные решения приведены только для случая с двумя продольными уголками жесткости. Показано, что когда продольные края зажаты, самые эффективные позиции для уголков жесткости являются на $0{\cdot}136d$ и $0{\cdot}284d$ от сжатого пояса, когда получается значение коэффициента продольного изгиба K равное 356. Когда продольные края свободно опираются, соответственные величины равны $0{\cdot}123d$, $0{\cdot}275d$ и 313. Для этих двух оптимальных условий были определены соотношения между параметрами γ и β уголка жесткости и относительного угла α. При значениях α ниже единицы было найдено, что когда оба уголка жесткости одинаковы, увеличение поддержки продольного края от простого шарнирного соединения к зажимному соединению влияет очень мало на жесткость уголка жесткости, нужного для того что бы получить максимальное возможное сопротивление к продольному изгибу. С другой стороны, для величин α превышающих единицу, влияние поддержки продольного края становится прогрессивно более важным.